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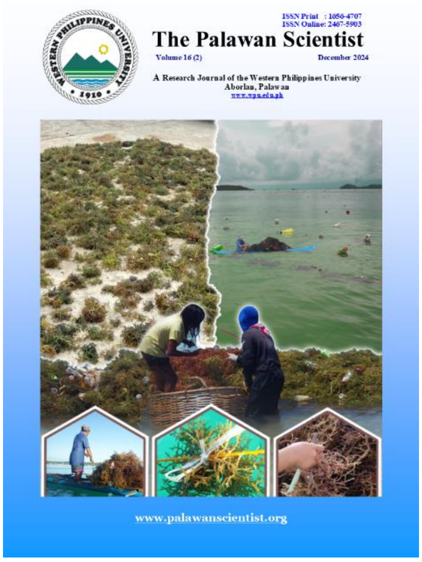

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Preservice teachers' proficiency in fraction subconstructs as predictors of conceptual understanding in fraction arithmetic

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ABSTRACT

The study aimed to determine whether the Kieren-Behr model holds true when examining the relationship between knowledge of fraction subconstructs and conceptual understanding of fraction arithmetic. Specifically, the study argued that the proficiency of pre-service teachers in fraction subconstructs can contribute to the development of conceptual knowledge in fraction arithmetic. It asserted that performance in problem-posing tasks, which reflect conceptual understanding, is significantly related to proficiency in different subconstructs of fractions. The proficiency of pre-service teachers in fraction subconstructs and their problem-posing performance were assessed using the expert-validated Fraction Subconstruct Test (FST) and Problem-Posing Test (PPT). The collected data were analyzed using descriptive statistics and standard multiple linear regression. Overall, the pre-service teachers only achieved a "beginning level" of proficiency in fraction subconstructs and performed unsatisfactorily in the PPT. Their proficiency in the measure subconstruct predicted conceptual understanding of adding fractions; their proficiency in the quotient subconstruct predicted conceptual understanding of subtracting fractions; their proficiency in the operator and quotient subconstructs predicted conceptual understanding of multiplying fractions; and their proficiency in the part-whole subconstruct predicted conceptual understanding of dividing fractions. The study suggests that teacher education institutions should develop intervention and enrichment programs to enhance the numerical competency of pre-service teachers, particularly in fractions. Additionally, curriculum writers are encouraged to emphasize mastery of each fraction subconstruct in order to promote successful development of conceptual understanding.

Keywords: education, mathematics, teaching

INTRODUCTION

The teachers' weak conceptual understanding of fractions is manifested in their poorly structured word problems. Consequently, this lack of understanding will burden the teaching-learning process when teachers are faced with contextual problems. Moreover, as teachers are entrusted with the role of developing their students to be good "problem posers," chances are, this aspect is being neglected in mathematics classrooms due to the inefficacy of posing quality problems to engage the students in a higher-order learning experience. This problem is prominent in literature (Copur-Gencturk 2021), specifically in fractions arithmetic, where most teachers and pre-service teachers fail to grasp its procedural and conceptual nature (Nillas, 2003; Lee and Lee 2023; Perry 2023; Tossavainen and Johansson 2023).

The difficulties in learning fractions can be attributed to the fact that fractions can have multiple meanings. A growing body of research suggests that fractions comprise a multifaceted construct, and each contributes to the student's proficiency in fraction arithmetic (Charalambous and Pitta-Pantazi 2005, 2007; Baker et al. 2012). For instance, Kieren (1980) proposed that fractions should be conceptualized as a set of interrelated constructs: the part-whole, ratio, operator, quotient, and measure subconstruct. He argues that exposure to numerous rational number subconstructs is necessary to fully understand fractions. Later, Behr et al. (1983) extended Kieren's (1980) ideas on fraction subconstructs by linking the different fraction subconstructs to the operations on fractions, fraction equivalence, and problem-solving procedures (Kieren-Behr model). The model (Figure 1) is hierarchical, with the part-whole subconstruct being the most basic subconstruct that is fundamental to understanding the other four subconstructs (ratio, operator, quotient, and measurement). The knowledge of the ratio, operator, and measurement subconstructs contributes to the understanding of equivalence, multiplication, and addition of fractions, respectively. All five subconstructs are essential for problem-solving. Charalambous and Pitta-Pantazi (2007) tested the hypothesis of the model on young learners (fifth and sixth graders), and Baker et al. (2009, 2012) tested it on adult learners (college students). Their studies found that adult and young learners have different fraction schemas and only provided partial support for the hypotheses of the model. It is important to note that the said model claims its hypothesis on operations with fractions solely under procedural competency.

However, more than procedural fluency is required for pre-service teachers to meet the demand for a great learning experience that promotes deeper learning of fraction operations. Hence, several studies have gone beyond the analysis of procedural fluency on fractions and have analyzed pre-service teachers' conceptual understanding through problem-posing tasks (Osana and Royea 2011; Kar and Işık 2014; Kilic 2015; Rosli et al. 2020). Although we agree that conceptual understanding cannot be directly measured and quantified, we can create opportunities for them to manifest this (Tichá and Hošpesová 2013; Cai and Hwang 2023). If supplemented by appropriate prompts, problem-posing tasks can be cognitively demanding activities requiring students' mastery of the concept to formulate solvable and real-life problems. The prompt is essential to force students to pose problems that are not easily solvable by known methods or just restatements of old problems with just changed givens; thus, problems posed by students can demonstrate their conceptual understanding (and misconceptions) as it is related to high math achievement and cognitive transfer (Donovan and Bransford 2005; Matsko and Thomas 2015).

With these arguments, the researchers of the present study subscribe to the assumption that proficiency in fraction subconstructs must have a significant relationship to the conceptual understanding of fraction arithmetic. Thus, considering the arguments above and the researchers' positionality, this paper will argue that pre-service teachers' knowledge of fraction subconstructs can also contribute to the development of conceptual knowledge in fraction arithmetic. This proposition can be supported by asserting that their performance in problem-posing tasks that manifest their conceptual understanding of fraction arithmetic significantly relates to their knowledge of the different subconstructs of fractions.

More specifically, this study aimed to address the following problems:

1. What are the pre-service teachers' level of proficiency in each fraction subconstruct, specifically the (a) part-whole, (b) measure, (c) operator, and (d) quotient subconstruct?

2. Which of the fraction subconstructs can be a predictor of conceptual understanding of (a) adding fractions, (b) subtracting fractions, (c) multiplying fractions, and (d) dividing fractions?

METHODS

Research Design

This study is descriptive and, at the same time, inferential as it uses a non-experimental predictive research design to address the research problems. The study used a descriptive research design, utilizing a questionnaire to describe the respondent's proficiency in fraction subconstructs, particularly on part-whole, measure, operator, and quotient. Thereafter, a regression analysis was followed to determine the significant predictors of the conceptual understanding of fraction arithmetic among the fraction subconstructs.

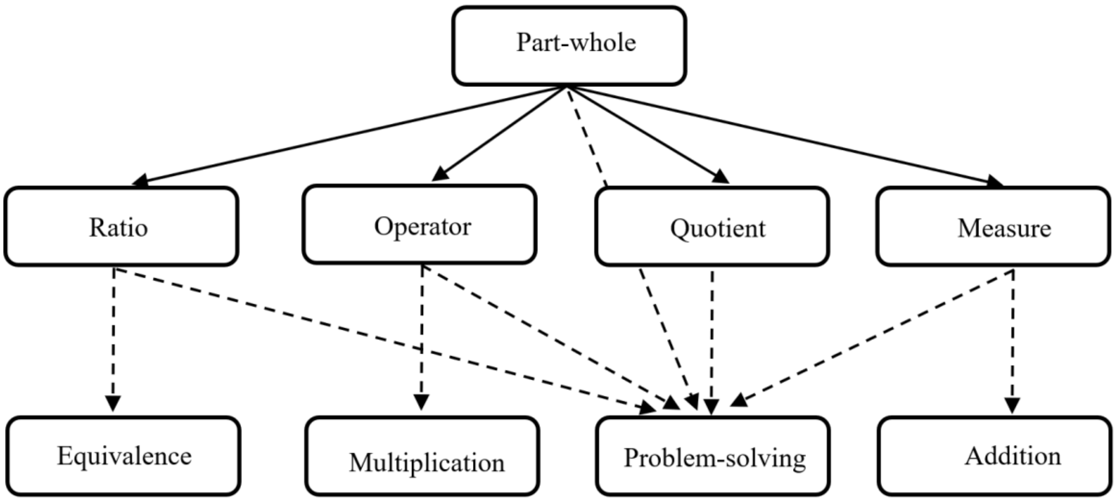


Figure 1. The Kieren-Behr Model (Behr et al. 1983).

Site and Participants

The respondents of the study were 61 pre-service teachers enrolled at a public university in central Luzon, Philippines, during the second semester of the academic year 2020-2021. This study used a purposive sampling design to select the respondents. Purposive sampling design is a sampling method in which the researcher uses his discretion to determine the respondents who best fulfill the study's objectives. The study purposefully chose the students enrolled in the Bachelor of Elementary Education program, as these pre-service teachers will soon be in-service teachers in the different municipalities or cities in Nueva Ecija and nearby provinces where elementary pupils' low mathematics achievement in the 2016-2017 National Achievement Tests is prevalent (Albano 2020). In addition, the sampling method was brought about by the restrictions of the Coronavirus (COVID-19) pandemic, in which the university was forced to go on fully online mode (asynchronous) classes, and random sampling would not be a practical option since only a limited number of students would have a reliable internet connection.

Data Collection

The researchers secured the necessary permission from the dean of the College of Education. Letters of courtesy and permission addressed to the above office have been circulated. The questionnaire distribution commenced after securing permission from the dean and the head of the elementary education department. The instrument was administered online through Google Forms on the dates set by the dean. The respondents were allowed to work independently for a time duration of 1 hour and 30 min.

Addressing the ethical issues in testing (Cohen et al. 2018), none of the respondents were forced to participate. The respondents answered the questionnaire with a complete understanding that the data collected would not be reflected in their academic records. Furthermore, the problem-posing tasks compelled them to work independently; unlike problem-solving, problem-posing does not have a single correct response. Thus, the researchers could easily discern if they had cheated. They scanned the students' responses and found that no two respondents had the same responses on the same item. The data's confidentiality and the respondents' anonymity were also assured, as the data is stored in a secured account, and no respondents' names were mentioned in the entire paper. Thus, no potentially delicate data will be traced back to each respondent.

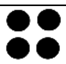
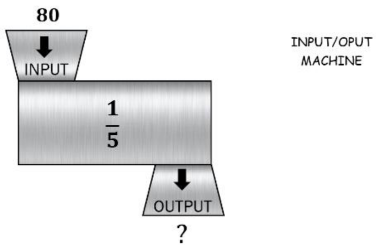
Instrumentation

The content validity of the Fraction Subconstruct Test (FST) and Problem-Posing Task (PPT) has been established by the two mathematics experts. A table of specifications was provided to help the experts examine the test instrument for the study. Several revisions have been made to the instruments to capture the overall purpose of the study.

Two test questionnaires were used in the study. The first test was the FST. The study excerpted and modified some test items used in Baker et al. (2012) and Charalambous and Pitta-Pantazi (2007) to measure the respondents' proficiency in FST. The FST is subdivided into four major parts that measure respondents' proficiency in each fraction subconstruct. The test for the part-whole subconstruct (7 items) contained three components: translating a picture to a symbolic fraction, translating symbolic fractions to the equivalent picture, and reconstructing the whole given a part. The test for the measure subconstruct (11 items) consists of identifying fractions as numbers and locating numbers on number lines, finding a number close to a certain number, comparing magnitudes of fractions, and finding a number between two numbers. The operator subconstruct test (5 items) consists of three components: representing mathematical statements into fractions, finding the output quantity given by the input and fraction operators, and finding the input quantity provided by the output and fraction operators. The test for the quotient subconstruct (6 items) consists of linking a fraction to the division of two numbers and solving partitive and quotative division situations. The second test is the PPT, which was divided into three sub-parts. Each sub-part was dedicated to each classification of problem-posing tasks (translating, comprehending, and selecting) described by Christou et al. (2005). Each of the three sub-parts consisted of four items, constituting the four operations on fractions (addition, subtraction, multiplication, and division). The decision to use the classification of Christou et al. (2005) is because the tasks are more structured (or at least semi-structured) in nature compared to the classifications proposed by Silver (1994) and Stoyanova and Ellerton (1996), which are "unstructured" or open-ended, which would be harder to score objectively and are more prone to bias.

A split-half method was utilized to test for the internal consistency of the FST. A Spearman-Brown coefficient of 0.825 has been computed, implying that the test is reliable. Tables 1 and 2 present the sample questions of the FST and PPT, respectively.

Table 1. Sample items of the Fraction Subconstruct Test.

Subconstruct	Sample Items
Part-whole	If these marbles  represent 2/3 of the whole set of marbles, draw the whole set of marbles.
Measure	Which of the following fractions is nearest to 1? a.) 1/2 b.) 2/3 c.) 4/5 d.) 5/6
Operator	The following diagram represents a machine that outputs 1/5 of the input number. What will be the output number if the input number is 80? 
Quotient	A 7-meter rope is to be cut into smaller pieces measuring 1/3 each. How many pieces can we cut from the rope?

The reliability of the scoring procedure was tested by asking two mathematics teachers to score 10% of the total problem-posing responses (cf. Tong et al. 2020). The inter-rater reliability coefficient was calculated using the formula described by Miles and Huberman (1994):

$$Inter - rater\ reliability = \frac{number\ of\ agreements}{number\ of\ agreements + number\ of\ disagreements}$$

As a result of applying the formula, a confidence percentage of 93% was obtained. The present study used a dichotomous scoring scheme for assigning values based on the respondents' response to the Fraction Subconstruct Test. Thus, the answers were given a numerical value of zero (0) if the response was incorrect and a numerical value of one (1) if the response was correct. The FST has a total score of 29 points. The allocation of items for each subconstruct was determined based on the K–12 Curriculum Guide for Mathematics.

For PPT, the study adopted and modified the scoring rubric from Cankoy and Özder (2017) to assess the respondent's performance in structured and semi-structured problem-posing tasks (Table 3). The rubric was originally intended to assess the problem-

Table 2. Sample items of Problem-Posing Test.

<table>
<tr><th>Type of PPT</th><th>Sample Item</th></tr>
<tr><td>Comprehending</td><td>Write an appropriate word problem based on the equation $\frac{1}{2}+\frac{1}{3}=n$.</td></tr>
<tr><td>Translating</td><td>Write a subtraction problem based on the table below.
<table>
<tr><th>Day</th><th>Cooked Rice</th></tr>
<tr><td>1</td><td>1/2 cavan of rice</td></tr>
<tr><td>2</td><td>1/3 cavan of rice</td></tr>
<tr><td>3</td><td>2/3 cavan of rice</td></tr>
<tr><td>4</td><td>1/3 cavan of rice</td></tr>
</table></td></tr>
<tr><td>Selecting</td><td>Write a question about the following story so that the answer to the problem is 5/6. Use the space provided after the story.
“Don Rafael ate 1/6 of a bibingka for a snack and 2/3 of the same bibingka for lunch. ____?</td></tr>
</table>

Table 3. The Problem-Posing Task Scoring Rubric. [a] For comprehending tasks only; [b] For translating tasks only; [c] For selecting tasks only.

CATEGORY	SUB-CATEGORY	EXPLANATION	SCORE
Solvability	Solvable	The information given in the problem is sufficient to solve the problem and find the solution.	1
	Unsolvable	The information given in the problem is not sufficient to solve the problem and find the solution.	0
Reasonability	Reasonable	The problem and the solution are reasonable and applicable in real life.	1
	Unreasonable	The problem and the solution are not reasonable and applicable in real life.	0
Language	Correct Mathematical Terms	The mathematical terms are correctly used in the problem	1
		The mathematical terms are not appropriately used in the problem	0
	Obeying grammar rules	The problem obeyed the grammar rules at all to express the question.	1
		The problem partly obeyed or did not obey grammar rules at all when expressing the question.	0
Restrictions	Appropriate to the given equation[a]	The solution to the problem fits the given equation	1
		The solution to the problem does not fit the given equation	0
	Appropriate to the given table[b]	The quantitative information on the table was used properly	1
		The quantitative information on the table was used properly	0
	Appropriate to the given situation and expected solution[c]	The problem satisfies the given situation, and the needed answer	1
		The problem satisfies the given situation, and the needed answer	0

posing performance of students in a free situation (PPT). The rubric was modified by removing two original categories that do not seem appropriate to assess structured and semi-structured PPTs (cf. Cankoy and Özder 2017). One category is also added to assess their problem-posing performance in specific problem-posing processes described by Christou et al. (2005). The PPT has a total score of 60 points. Each of the operations is given a maximum score of 15 points.

RESULTS

Pre-service Teachers' Proficiency in Fraction Subconstructs

The pre-service teachers performed unsatisfactorily in the FST (Overall MPS $= 51.76$) (Table 4), suggesting they have about 52% mastery of the expected competencies. Furthermore, the computed average standard deviations (SD = 25.98) of scores indicate that the Elementary Pre-service Teachers' scores range from beginning to proficient levels (51.76 ± 25.98).

Elementary Pre-service Teachers' Problem-Posing Performance in Fraction Arithmetic

They performed unsatisfactorily in the PPT (Overall mean = 2.00), suggesting that their conceptual understanding of fraction arithmetic is not fully developed (Table 5). The computed overall SD indicates that student performance has a wide variability, ranging from poor to excellent (2.00 ± 3.25).

Table 4. Pre-service teachers' proficiency in fraction subconstructs. Note: Unsatisfactory/Beginning = 0.00-59.99; Fairly Satisfactory/Developing = 60.00-67.99; Satisfactory/Approaching Proficiency = 68.00-75.99; Very Satisfactory/Proficient = 76.00-83.99; Outstanding/Advanced = 84.00-100.00. MPS – mean percentage score_; SD – standard deviation.

FRACTION SUBCONSTRUCTS	MPS	SD	DESCRIPTION
Part-whole subconstruct	**64.87**	**25.09**	**Fairly Satisfactory / Developing**
1. Translating the picture to a symbolic fraction	75.41	26.18	Satisfactory / Approaching Proficiency
2. Translating symbolic fractions to picture	34.43	47.91	Unsatisfactory / Beginning
3. Reconstructing the whole given a part of it	42.62	49.86	Unsatisfactory / Beginning
Measure subconstruct	**39.49**	**20.25**	**Unsatisfactory / Beginning**
1. Identifying fractions as numbers	65.57	47.91	Fairly Satisfactory / Approaching Proficiency
2. Locating numbers on number lines	36.07	24.72	Unsatisfactory / Beginning
3. Comparison of magnitudes of fractions	50	37.64	Unsatisfactory / Beginning
4. Finding a number between two fractions	24.59	43.42	Unsatisfactory / Beginning
5. Finding a number closer to one	31.98	39.83	Unsatisfactory / Beginning
Operator subconstruct	**47.21**	**27.58**	**Unsatisfactory / Beginning**
1. Representing mathematical statements into fractions	63.93	29.01	Fairly Satisfactory / Approaching Proficiency
2. Finding output quantity given input and fraction operator	41.8	44.89	Unsatisfactory / Beginning
3. Finding input quantity given output and fraction operator	24.59	43.42	Unsatisfactory / Beginning
Quotient subconstruct	**55.46**	**23.91**	**Unsatisfactory / Beginning**
1. Linking a fraction to the division of two numbers	75.41	43.42	Satisfactory / Beginning
2. Partitive division	47.54	30.10	Unsatisfactory / Beginning
3. Quotative division	57.38	40.66	Unsatisfactory / Beginning
Overall MPS	**51.76**	**25.98**	**Unsatisfactory / Beginning**

Table 5. The respondents' problem-posing performance in operations with fractions. Note: Poor = 0-1.25; Unsatisfactory = 1.26-2.50; Satisfactory – 2.51-3.75; Excellent = 3.76-5.00.

OPERATION	x̄	SD	DESCRIPTION
Addition	**2.44**	**3.69**	**Unsatisfactory**
Comprehending	2.41	1.99	Unsatisfactory
Translating	3.02	1.88	Satisfactory
Selecting	1.89	1.46	Unsatisfactory
Subtraction	**1.95**	**3.58**	**Unsatisfactory**
Comprehending	1.95	1.91	Unsatisfactory
Translating	1.33	1.63	Unsatisfactory
Selecting	2.56	173	Satisfactory
Multiplication	**1.70**	**3.82**	**Unsatisfactory**
Comprehending	1.08	1.49	Poor
Translating	1.69	1.77	Unsatisfactory
Selecting	2.33	1.74	Unsatisfactory
Division	**1.90**	**4.13**	**Unsatisfactory**
Comprehending	1.61	1.88	Unsatisfactory
Translating	2.26	1.68	Unsatisfactory
Selecting	1.82	1.91	Unsatisfactory
Overall Performance	**2.00**	**3.25**	**Unsatisfactory**

Predictors of Conceptual Understanding of Arithmetic Operations on Fractions

When examining the predictors of conceptual understanding of arithmetic operations on fractions, multiple linear regression analyses were conducted. The Variance Inflation Factor (VIF) of the independent variables ($\text{part}-\text{whole} = 1.66, \text{measure} = 1.30, \text{operator} = 1.04, \text{and quotient } 1.49$) was less than 2 in all models, indicating no severe multi–collinearity among the independent variables. Furthermore, the tests for normality of the residuals using the Shapiro-Wilk test and the Durbin-Watson Test for autocorrelation were not significant ($P > 0.05$).

The results revealed that EPTs' proficiency in the measure subconstruct significantly predicts their conceptual understanding of adding fractions (Table 6). Additionally, their proficiency in the quotient subconstruct predicts their conceptual understanding of subtracting fractions (Table 7). Moreover, their proficiency in operator and quotient subconstruct predicts a conceptual understanding of multiplying fractions (Table 8), while their proficiency in the part-whole subconstruct predicts their conceptual understanding of dividing fractions (Table 9).

Table 6. Predictors of conceptual understanding of adding fractions. Note: * Test is significant at the 0.05 level (2-tailed); ***Test is significant at the 0.001 level (2-tailed).

Predictor	Coefficient	Standard Error	*t*-value	*p*-value
Part-Whole Subconstruct	0.182	0.305	0.596	0.554
Measure Subconstruct	0.445	0.213	2.089	0.041*
Operator Subconstruct	0.394	0.308	1.277	0.207
Quotient Subconstruct	0.649	0.345	1.831	0.072
Constant	1.465	1.360	1.077	0.286
R-square = 0.286 Multiple R = 0.535 F (4,56) = 5.62***				

Table 7. Predictors of conceptual understanding of subtracting fractions. Note: * Test is significant at the 0.05 level (2-tailed); ***Test is significant at the 0.001 level (2-tailed).

Predictor	Coefficient	Standard Error	*t*-value	*p*-value
Part-Whole Subconstruct	0.442	0.305	1.474	0.146
Measure Subconstruct	-0.095	0.210	-0.453	0.652
Operator Subconstruct	0.465	0.304	1.531	0.131
Quotient Subconstruct	0.792	0.349	2.269	0.027*
Constant	0.508	1.339	0.379	0.706
R-square = 0.266	Multiple R = 0.516	F (4,56) = 5.08***		

Table 8. Predictors of conceptual understanding of multiplying fractions. Note: * Test is significant at the 0.05 level (2-tailed); **Test is significant at the 0.01 level (2-tailed).

Predictor	Coefficient	Standard Error	*t*-value	*p*-value
Part-Whole Subconstruct	0.171	0.326	0.524	0.602
Measure Subconstruct	0.004	0.227	0.019	0.985
Operator Subconstruct	0.820	0.329	2.489	0.016*
Quotient Subconstruct	0.767	0.378	2.028	0.047*
Constant	-0.183	1.452	-0.126	0.900
R-square = 0.243	Multiple R = 0.492	F (4,56) = 4.48**		

Table 9. Predictors of conceptual understanding of dividing fractions. Note: * Test is significant at the 0.05 level (2-tailed); **Test is significant at the 0.01 level (2-tailed).

Predictor	Coefficient	Standard Error	*t*-value	*p*-value
Part-Whole Subconstruct	0.797	0.350	2.278	0.027*
Measure Subconstruct	-0.249	0.244	-1.018	0.313
Operator Subconstruct	0.268	0.354	0.756	0.453
Quotient Subconstruct	0.725	0.407	1.781	0.080
Constant	0.105	1.562	0.067	0.947
R-square = 0.249	Multiple R = 0.499	F (4,56) = 4.64**		

DISCUSSION

Proficiency in Fraction Subconstructs

The results of the descriptive analysis demonstrate that most pre-service teachers were proficient in the part-whole subconstruct. This suggests that understanding the part-whole interpretation of fractions is easier for pre-service teachers compared to other aspects of fractions. This finding supports the Kieren-Behr model and implies that the part-whole interpretation is fundamental to learning fractions. The results also indicate that pre-service teachers are comfortable interpreting fractions from visual representations. This verifies the popular notion that it is easier to learn with visual aids, similar to what Mendiburo et al. (2014) found in learning part-whole through visual aids. The students must first understand the part-whole concept of the fraction before moving on to understanding the other four subconstructs. Nevertheless, this is also aligned with the conclusion of the study concluded by Baker et al. (2009) and Charalambous and Pitta-Pantazi (2007). The part-whole subconstruct being the most straightforward interpretation to acquire is not surprising since it is the most common representation of fractions starting from primary school (Alajmi 2012; Kolar et al. 2018; Jiang 2021).

To develop a robust understanding of the measure subconstruct, an understanding of portioning and the density property of rational numbers is required, as the number of fractions between any two fractions is infinite (Charalambous and Pitta-Panttazi 2007). The overall low proficiency in the measure subconstruct can be attributed to pre-service teachers' difficulties dealing with number lines, especially when asked to locate a point on a number line (Widjaja et al. 2011; Ergene and Ergene 2020; Jiang 2021). Furthermore, Tunc-Pekkan (2015) found that students performed poorly on problems with number lines compared to other graphical representations.

For the operator subconstruct, pre-service teachers also demonstrated unsatisfactory performance. This aligns with the notion that understanding fractions as operators, involving multiplication and division operations is challenging due to their multiplicative nature (Kieren 1976). Similar findings were reported by Buforn et al. (2017), highlighting pre-service teachers' difficulties in recognizing students' reasoning with inverse fraction operators.

The pre-service teachers also performed poorly in the quotient subconstruct, indicating a lack of recognition that any fraction can be seen as the result of a division. Specifically, they struggled with discerning that the fraction *x/y* denotes the numerical value obtained when *x* is divided by *y*. This deficiency can be attributed to their unsatisfactory performance in quotative and partitive divisions. The result suggests that the pre-service teachers are not yet proficient in the "repetitive subtraction" and the "fair-sharing" concept of division. On some note, the results contradict the study conducted by Clarke and Roche (2009), who concluded that pre-service teachers were more successful in solving partitive division than quotative division. However, they partially support the findings of Lee (2017), who found that most pre-service instructors could perform procedural fraction division calculations but had limited understanding of the quantitative meaning of quotative division.

Overall, the pre-service teachers' proficiency in fraction subconstructs fell short of the 50% mastery expected. This result is consistent with previous studies that revealed limited and unsatisfactory knowledge of fractions among pre-service teachers (Van Steenbrugge et al. 2013; Avcu 2019). Similarly, Lee et al. (2015) suggested that many pre-service teachers have limited understanding of fraction subconstructs, particularly in the measure and operator subconstructs.

Problem-Posing Performance in Fraction Arithmetic

Generally, the pre-service teachers performed unsatisfactorily on the problem-posing task. This indicates that they posed problems that captured only limited aspects of the different semantic structures of the operation. Akay and Boz (2009) commented that pre-service teachers who did not have the opportunity to pose problems during their university years would encounter difficulties in preparing and posing practical problems for their students. They would likely rely mostly on textbook problems, giving assessments that are not sensitive to the student's level of understanding.

The unsatisfactory performance of the pre-service teachers in posing addition problems in the comprehending-type PPT suggests that they face difficulties in making sense of the given equation by translating mathematical expressions into verbal expressions. Similarly, they performed unsatisfactorily on the selecting–type of PPT when adding fractions. This also shows that pre-service teachers need help with the operational and semantic structure of adding fractions. On the other hand, the pre-service teachers performed satisfactorily on translating-type PPT by adding fractions, indicating that they could translate quantitative information embedded on a table into a verbal mathematics problem. Overall, the data revealed the deficiency of the pre-service teachers in problem-posing performance in adding fractions. This supports the study of Dogan-Coskun (2019), who further asserted that although the pre-service elementary teachers were able to pose problems focusing on the "part-part-whole" or "joining" meanings of the addition operation, most of them posed problems with at least one error.

The unsatisfactory performance of the pre-service teachers in posing subtraction problems in the comprehending-type PPT suggests that they need help in formulating verbal mathematics problems as they could not grasp the overall meaning of the equation. Similarly, they performed unsatisfactorily on the translating type of PPT in subtracting fractions. This also shows that pre-service teachers have problems translating quantitative information into tabular form. In contrast, the pre-service teachers performed satisfactorily in the selecting-type of PPT in subtracting fractions. This indicates that they can translate quantitative information into a verbal problem in subtracting fractions. Similar results were articulated by the study conducted by Dixon et al. (2014), where they found that most pre-service teachers could not pose a problem with subtraction problems appropriately. Furthermore, they found that pre-service teachers, when asked to pose a problem presenting the expression $a-b$, tend to pose problems representing $a-(ab)$, especially in the domain of fractions because of incorrect redefinition of the whole. Similar errors were noticed in the problems posed by the pre-service teachers in the present study.

The results suggest that the pre-service teachers have difficulty posing a multiplication problem in all three types of PPT, especially in the comprehending-type of task. This further suggests that they could not wholly translate equations, situations, tables, and figures into verbal mathematics problems when multiplying fractions. In total, this result was comparable to the analysis of Luo (2009), where their analysis suggests that a significant percentage of the pre-service teachers in the study could not construct an entirely appropriate fraction multiplication word problem, and their known semantic structures are limited. Based on these results, Luo (2009) asserted that pre-service teachers should at least possess both mathematical content knowledge and problem-posing skills.

From the results, it can also be concluded that they performed worse in posing multiplication problems compared to the other operations. This contradicts the common knowledge that division is more complicated to grasp due to its more complex structure (Tirosh 2000). Moreover, this contradicts the findings of Xie and Masingila (2017), who concluded that division was the most challenging operation for posing story problems among the four operations. This further indicates that pre-service teachers were not familiar with or comfortable with the semantic structures of multiplication problems (e.g., part-of-a-whole and area). It can also be noted from the results of the present study that the pre-service teachers performed poorly on the comprehending-type of PPT. These difficulties in the transformation of symbolic expressions into verbal expressions are noticeable at the international level (Kar and Işık 2014). These alarming results suggest that formal training and frequent exposure to problem-posing tasks, especially in the multiplication-comprehending-type, need attention from teacher educators and authorities.

The unsatisfactory performance of the pre-service teachers in posing division problems conforms with the study conducted by Leung and Carbone (2013). Their analysis showed that only one-third of pre-service teachers could pose reasonable division problems that require a fraction divisor. Thus, in conjunction with their analysis, this depicts that many pre-service teachers do not have a rigorous understanding of the meaning of "dividing by a fraction." They could potentially skip explanations of these concepts with their future students or just focus on computation procedures.

Predictors of Conceptual Understanding of Arithmetic Operations on Fractions

To recall, the study aimed to argue that the fraction subconstruct must be able to predict not only procedural fluency in fraction arithmetic but also conceptual understanding. For comparison, Figure 1 presents the Kieren-Behr model, and Figure 2 summarizes the results of the series of regression analyses**.** We can notice that the analyses partially concurred with the hypothesis of the Kieren-Behr model except for the quotient subconstruct.

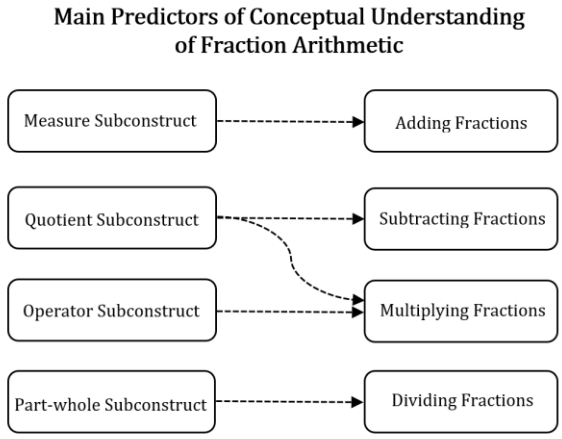


Figure 2. Summary of the regression analysis.

For predicting of the conceptual understanding of adding fractions, it was found that proficiency in the measure subconstruct is the most critical predictor. Pre-service teachers with a firm grasp of the measure subconstruct are likely adept at recognizing and manipulating fractions in various contexts, enabling them to better conceptualize the addition of fractions. The measure subconstruct, encompassing numerical relationships and magnitude comprehension, provides a foundation for pre-service teachers to accurately assess and perform operations related to the quantitative aspects of fractions. The result is consistent with the hypothetical pathways of the Kieren-Behr model. Furthermore, it supports the analysis of Baker et al. (2009) on adult learners, in which they found that understanding the measure subconstruct implies proficiency in adding fractions. The result suggests that teacher educators should pay attention to and plan intervention on pre-service teachers' proficiency in determining magnitudes of fractions, locating numbers on number lines, finding a number between two fractions, and finding a number closer to one, as they showed a lack of understanding in these areas. It has been shown that this lack of understanding may contribute to their development of concepts in adding fractions.

The quotient subconstruct appeared to be the primary predictor of the conceptual understanding of subtracting fractions. The result disagreed with the analysis of Charalambous and Pitta-Pantazi (2007) and Baker et al. (2009), who suggested that the measure subconstruct must also have a significant relationship with solving subtraction problems. However, the result of the present study showed that when it comes to posing subtraction problems, the relationship between the measure subconstruct and the operation of subtraction does not hold. A plausible reason for the relationship found between the quotient subconstruct and the conceptual understanding of subtracting fractions is that pre-service teachers may be using their proficiency in "repeated subtraction" (quotative division) as a strategy for building relationships between the given quantitative information on each problem in subtraction. Nevertheless, the result of the analysis implies that the proficiency of the pre-service teachers in solving quotative and partitive division problems must be emphasized in the curriculum to help them be better problem-solvers when it comes to subtracting fractions.

For the conceptual understanding of multiplying fractions, the operator and quotient subconstruct appear to be the main predictors. The result is consistent with the hypothesis of the Kieren-Behr model, which suggests that knowledge of the operator subconstruct implies procedural knowledge of multiplying fractions. It is important to note that the quotient subconstruct was also found to be a significant predictor, which is comparable with the study of Charalambous and Pitta-Pantazi (2007) and Baker et al. (2009) where they found that knowledge of the quotient subconstruct is more important than the operator subconstruct in predicting fluency in multiplying fractions both young and adult learners. A plausible explanation for their relationship is that pre-service teachers may be using their proficiency in the "invert-multiply strategy" (a strategy commonly used to solve division problems) to establish relationships between the quantitative information needed to pose multiplication problems involving fractions. The results suggest that teacher educators should pay attention to and plan interventions for pre-service teachers' knowledge of partitive and quotative divisions as these help understand problem-posing tasks related to fraction multiplication.

Surprisingly, the quotient subconstruct did not significantly predict the concept of dividing fractions. Results showed that proficiency in the part-whole subconstruct is the main predictor of the conceptual understanding of dividing fractions. This result supports the relationship between part-whole and division found by Bicknell and Loveridge (2015). This further implies that pre-service teachers might be using the part-whole concept, in which the parts are considered components of the whole, to help identify which quantitative information serves as the dividend (part) and which one serves as the divisor (whole).

Recommendation for Instruction and Future Researchers

The significant regression model suggests that fractions subconstructs should be introduced first before the concepts of adding, subtracting, multiplying, and dividing fractions. These subconstructs will serve as a stepping stone in successful assimilating and accommodating the semantic structures of the corresponding fraction operations.

Due to low performance in both FST and PPT, we encourage curriculum writers and educators of pre-service teachers to revisit the mathematics curriculum offered to them and consider providing an adequate learning episode for fraction subconstructs.

A separate study dedicated solely to the detailed description of the mental activities and misconceptions of pre-service teachers on operations with fractions manifested through problem-posing tasks and a follow-up interview is highly recommended for future researchers. Furthermore, the analysis of the creativity of the problem-posers may be given a spot light to provide a more explicit role for proficiency in fraction subconstructs.

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ETHICAL CONSIDERATIONS

The dean approved the letter to conduct the study inside the College of Education, Central Luzon State University. Consent from each respondent was also considered. No traceable data was collected from the respondents; thus, the confidentiality of the responses is secured.

DECLARATION OF COMPETING INTEREST

The authors declare that there are no competing interests from any authors.

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