

A unified stochastic framework with memory for heat index and sea level dynamics

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ABSTRACT

Monitoring temperature-dependent events is critical for understanding their dynamics since these events have an impact on both animal and human habitation. It is common to see analysis of heat index and sea level that are described separately although these events have a direct connection to temperature. Often these analyses are less effective and less reliable in describing its dynamics vis-à-vis redundancy, flexibility, accounting of uncertainties and optimization. Since both are temperature-dependent events, a unified stochastic model with memory was derived. These events can be effectively described with a collective memory function $(T - t)^{\frac{\mu-1}{2}} e^{-\frac{\beta}{2t} t^{\frac{\mu+1}{2}}}$, modifying the Brownian motion. A good match between the empirical and theoretical MSDs for both heat index and sea level was obtained with memory parameters $\mu_{HI} = 1.0460$ and $\mu_{SL} = 1.0894$, respectively. With $\mu > 1$, heat index and sea level exhibited long-term memory characteristics which have important implications for large timescale prediction. Similarly, analyses using a unified model are simplified and may provide the interrelatedness of these events.

Keywords: collective memory function, forecasting, heat index, non-Markovian, Philippines, sea level

INTRODUCTION

The unprecedented rise in global temperature at a rate of 0.2 degrees/decade over the past thirty years (Hansen et al. 2006) had significantly affected and disrupted both human activities and animal habitation. Among the many issues of elevated

temperature, the most pressing is the health-related heat stresses (Dang et al. 2019) including effect on respiratory systems and the weakening of body to maintain temperature balance (Ma et al. 2019; McGregor and Vanos 2018). The elevated island



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heating, as measured by the heat index (HI), has the greatest impact on human productivity (Wang et al. 2022; Dean and Claassen 2023), while animal habitation decreases as sea level (SL) rises (Paul and Paul 2022; Dixon et al. 2023). In many ways, these observed effects are commonly related to temperature rise. Note that changes in SL is related to rising temperature causing ice melts in both northern and southern hemisphere (Hagen and Azevedo 2022; Orr et al. 2022; Coupe et al. 2023; Park et al. 2023; Purich and England 2023). It is but important to constantly monitor the state of these climactic factors especially in areas that are highly vulnerable to these changes particularly at their extreme state (Balacco et al. 2023; Zagebelnaya 2022).

Since HI and SL are both temperature-dependent events, they may exhibit a similar dynamical model. Because of this, a separate treatment may provide inconsistencies (Meehl et al. 2000; Pielke et al. 2002; Li et al. 2010) and non-conformities of the models resulting to less reliable assessments of current extreme events (Eggleston et al. 2006; Long et al. 2006; Field et al. 2012; Stott et al. 2016). In fact, HI and SL model unification stems from the fact that both phenomena evolve in time can be linked to the same physical processes that may have operated in multiple spatial and temporal scales (Brown et al. 2012). The vantage point is the ability of a unified model for a seamless prediction for these systems (Bhaskaran et al. 2002). For example, the novel Met Office Unified Model for climate change and weather prediction had proven to address model issues on redundancy, flexibility and often outperforms stand-alone models (Brooke et al. 2019; Maher and Earnshaw 2022). On the other hand, it had been shown in Elnar et al. (2021) that interrelated dynamics can have unifying models highlighting similar memory functions with varying degrees. With the interrelatedness between HI and SL, the researchers are driven to demonstrate that these events do, in fact, follow a similar dynamical memory parameter. By employing the analytical stochastic framework with memory (Bernido and Carpio-Bernido 2012, 2014), a direct comparison of the

analytical and empirical mean square deviations (MSD) is provided and we obtain the explicit probability distribution function (PDF). In this way, a unified treatment can provide a holistic perspective in the interrelatedness of temperature-dependency of these biophysical events.

METHODS

Stochastic Framework with Memory

The non-Markovian structure of fluctuations of temperature-dependent phenomena such as heat index (HI) and sea level (SL) was modeled using the Hida stochastic functional integral method (Hida 1996). This method enables the researchers us to analyze the PDF and moments analytically. The parametrization of the path of a random variable x was defined as a sum of the initial point and fluctuation (Equation 1; Bernido and Carpio-Bernido 2012, 2014), where $B(t)$ is the ordinary Brownian motion, $f(T - t)h(t) =$

$(T - t)^{\frac{\mu-1}{2}} e^{-\frac{\beta}{2t}} t^{\frac{\mu+1}{2}}$ is a memory function, and $g(T)$ is a modulating factor in $F(T)$. To pin down a particular trajectory of interest, the delta function constraint was applied, $\delta(x(T) - x_T)$ and the probability distribution function (PDF) was obtained for the given endpoint x_T by taking the expectation value of the delta function. The corresponding PDF is given by Equation 2 and $\alpha^2 = (g(T))^2 \left(\int_0^T \left[(T - t)^{\frac{\mu-1}{2}} e^{-\frac{\beta}{2t}} t^{\frac{\mu+1}{2}} \right]^2 dt \right)^{-1}$. From the PDF, we can obtain

the mean square displacement (MSD) as given by Equation 3. Notice that if $f(T - t) = \sqrt{2D}$, $h(t) = g(T) = 1$, the PDF and MSD correspond to the ordinary Brownian motion. The analyses used in this research were applied to temperature-related occurrences, with a focus on sea level (SL) and heat index (HI). Datasets of these two events are readily available and can be accessed freely through NOAA and NWS websites as described in the next section.

$$x(T) = x_0 + F(T) = x_0 + g(T) \int_0^T (T - t)^{\frac{\mu-1}{2}} e^{-\frac{\beta}{2t}} t^{\frac{\mu+1}{2}} dB(t) \tag{Eq.1}$$

$$P(x_T, T; x_0, 0) = \left(\frac{2\pi}{\alpha^2} \right)^{\frac{1}{2}} \exp \left(-\alpha^2 \frac{(x_0 - x_T)^2}{2} \right) \tag{Eq.2}$$

$$MSD = g(T)^2 \int_0^T [f(T - t)h(t)]^2 dt. \tag{Eq.3}$$

Heat Index Dataset

Heat index data were gathered from the National Oceanic and Atmospheric Administration (NOAA) data site in 2018 from 1966 to 2017, totaling 17,588 daily HI data points. In this case, HI datasets obtained were for Mactan Island, Cebu. These HIs reflected the US National Weather Services (NWS) algorithm (Anderson et al. 2013) which provided consistency to environmental results and agreed among the many algorithms to Steadman’s apparent temperature (Anderson et al. 2013; Ramirez-Beltran et al. 2017). With a huge amount of data points, Leskovec and Faloutsos (2006) suggested that a subgraph can be surveyed of which 25% of the data points may observe a similar behavior as that of the original graph. Thus, the corresponding subgraphs were plotted and compared to whether they exhibited the same behavior. Only then the representative 25% of the total data points were used in the model. A linear interpolation approach was used in filling the missing data points as it was done in Bucheli et al. (2022). The heat index fluctuation for Mactan Island is presented in Figure 1.

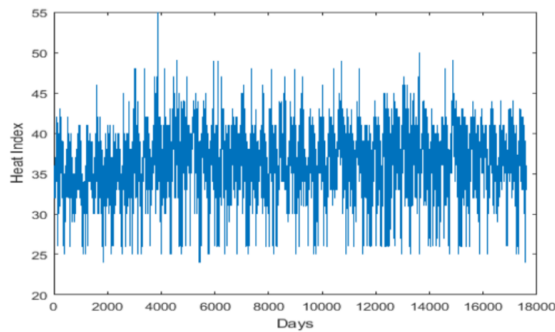


Figure 1. Daily fluctuations of heat index in Mactan Island from 1966 to 2017.

Sea Level Datasets

The SL data was obtained from the database of the University of Hawaii Sea level center for the Philippines, particularly the Manila Sea level data points. The period covered in the dataset is from 1984 to 2015. For consistency, a 25% representative of the total data points were used in the model. Similarly, missing data points were filled in using linear interpolation (He et al. 2022; Zheng et al. 2022). The fluctuations of SL for Manila are presented in Figure 2, depicting the original data that comprises both deterministic and stochastic components. In order to isolate the purely stochastic part of the data, detrending techniques were applied, and the resulting plot is presented in Figure 3.

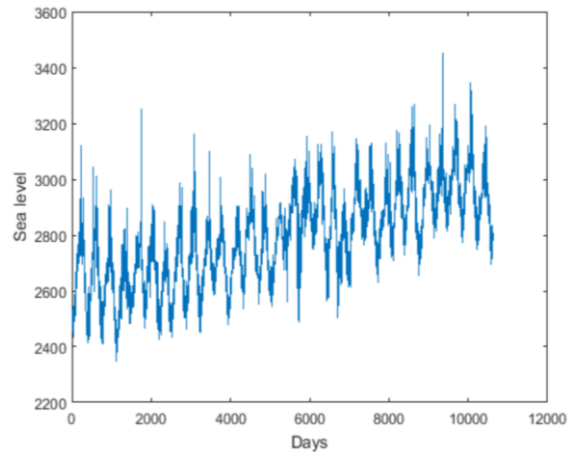


Figure 2. Daily sea level (cm) for Manila, Philippines (1984–2015).

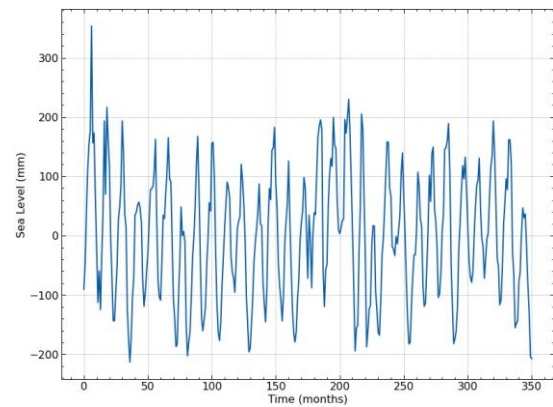


Figure 3. Detrended raw data plot showing long-term fluctuations in sea level over time.

Mean Square Displacement (MSD) Plots

MATLAB R2018b was used in obtaining the corresponding plots of HI and SL and the plots of their MSD’s. The MSD plots were fitted against the theoretical MSD. This theoretical MSD was chosen appropriately to give a good fit to the empirical MSD. Lastly, the corresponding parameters were obtained from this comparison.

RESULTS

Stochastic Framework for HI and SL

The different theoretical MSD’s describing stochastic framework with memory was surveyed. The theoretical MSD is exponentially modified for both events and is given by:

$$MSD = g(T) \left(\frac{\Gamma(\mu) t^{(\mu - 1)} e^{-\beta t}}{\beta^\mu} \right) \tag{Eq.4}$$

where $g(T) = \exp\left[\left(b - \frac{t}{\epsilon}\right)\sin(ct - k)\right]$ serves as the modulating function where b, ϵ, c and k are just constants. This is an extended form as used in Bernido et al. (2014). This theoretical MSD corresponded to the memory function given by $f(T - t) = (T - t)^{\frac{\mu-1}{2}}$, $h(t) = \frac{e^{-\beta/2t}}{t^{(\mu+1)/2}}$. Then, this stochastic model was applied and insights into the dynamics of the heat index and sea level were gained.

Empirical and Theoretical MSDs for HI and SL

Using the theoretical MSD above, the corresponding log-log plots for the empirical data alongside a theoretical fit of the HI datasets are presented in Figure 4. The corresponding coefficient of determination ($R^2 = 0.7309$) between the two is also provided to assess the quality of the fit. This comparison is essential to determine the accuracy of the theoretical model in describing the observed phenomenon. Moreover, as shown, the dynamical parameters derived from the fit corresponds to $\mu = 1.0460$ and $\beta = 0.0792$ with scaling constants of

the modulating function obtained as $b = 0.1377$, $c = 0.063$, $\epsilon = 9999$, and $k = 0.9$. Furthermore, the theoretical MSD with values of $t_c = 3.7$ (x-axis) were normalized. The normalization shifts the graph along x-axis.

The corresponding log-log plot of both the empirical and theoretical MSDs for sea level is presented in Figure 5 below. The corresponding coefficient of goodness of fit to be $R^2 = 0.9312$ was determined. From the fit, the parameters associated with the dynamics of the event were $\mu = 1.0894$ and $\beta = 1.4562$. The scaling constants of the modulating function were obtained as $b = 0.9129$, $c = 2.6569$, $\epsilon = 600$, and $k = 0.8$. Normalization of the x-axis had value of $t_c = 10^{0.5}$ which shifts the graph sideways in order for the two plots to match.

Henceforth, using Equation 4, the explicit form of the Probability Distribution Function of Equation 2 is expressed in Equation 5:

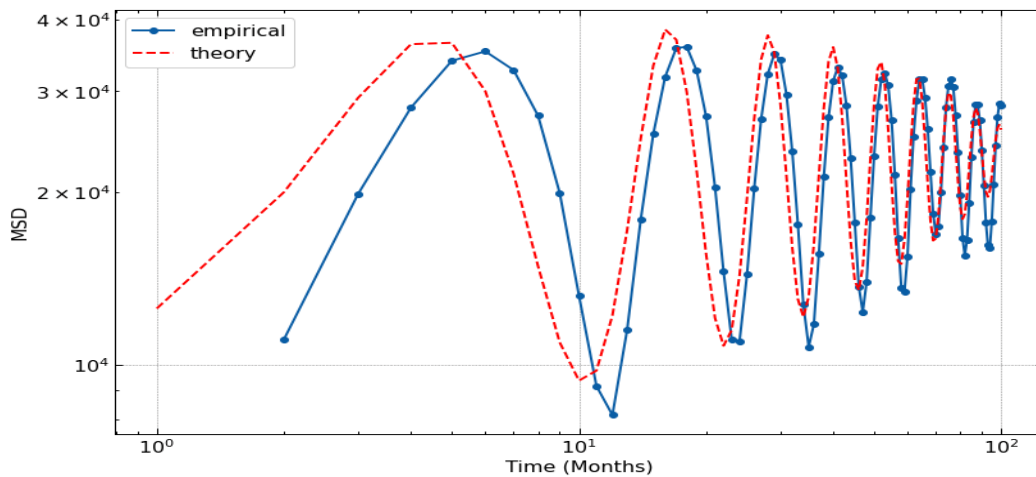


Figure 4. Log-log plots of empirical (blue) and theoretical (red) mean square displacement (MSDs) for heat index.

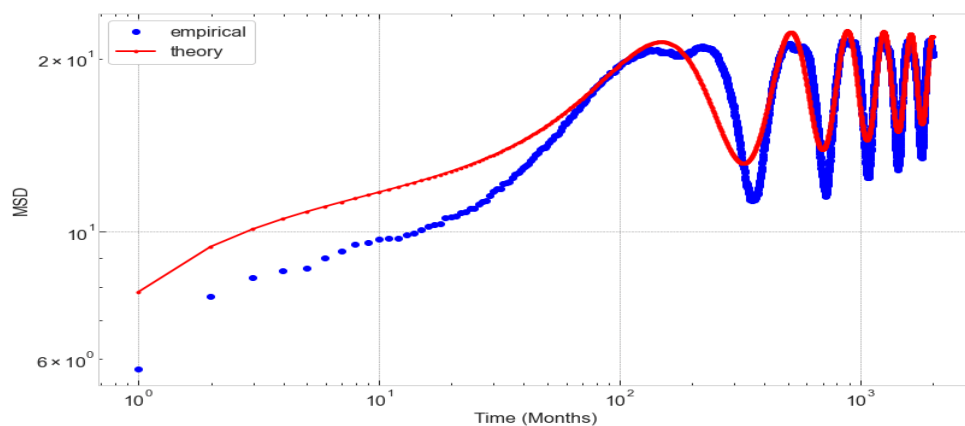


Figure 5. Log-log plots of empirical (blue) and theoretical (red) mean square displacement (MSDs) for sea level (SL).

$$P(x_T, T; x_0,) = \frac{\beta^\mu}{\sqrt{2\pi \exp\left[\left(b - \frac{T}{\epsilon}\right)\right] \sin(cT - k) \Gamma(\mu) T^{\mu-1} e^{\frac{\beta}{T}}}} \times \exp\left[-\frac{(x_T - x_0)^2}{2 \exp\left[\left(b - \frac{T}{\epsilon}\right)\right] \sin(cT - k) \Gamma(\mu) T^{\mu-1} e^{\beta/T}}\right] \quad (\text{Eq.5})$$

DISCUSSION

Stochastic Framework for HI and SL

The same stochastic framework for both HI and SL clearly demonstrated that they are, in fact, generally related phenomena. In context, this framework established a dynamical perspective of which both events can be described given that they are two distinct phenomena. Although both are driven by temperature changes, their dynamical behavior is crucial to the impacts because it influences other systems with which they interact. According to World Meteorological Organization (WMO 2021), they had shown the dynamical effects of heat and sea level on extreme weather events including cyclones, drought and wildfire.

It is noted, further, that the HI–SL interactions can be presumed as a driven dynamics resulting to a collective memory between systems as discussed in Elnar et al. (2021), in evo-eco dynamics (Power et al. 2015; Fisher and Pruitt 2020), eco-memory of interacting systems (Baho et al. 2021; Khaligli et al. 2021) or climate – carbon cycle interaction (Page et al. 2021). This collective memory is encoded in the characteristic parameter, μ , of Equation 4. This is anticipated to yield the same characteristic parameter as in the cases of HI ($\mu = 1.0460$) and SL ($\mu = 1.0894$).

Empirical and Theoretical MSDs for HI and SL

It is proven that both heat index and sea-level events have similar stochastic memory functions, as indicated in Equation 4. With the memory parameter $\mu = 1.0460$ HI and $\mu = 1.0894$ SL greater than unity suggesting long memory ranges which complemented the views of sea level as random fluctuations with memory (Peng et al. 1994; Li et al. 2011; Dangendorf et al. 2014; Ventosa-Santaularia et al. 2014; Elnar et al. 2021) and other temperature-dependent events, such as surface air temperature (Caballero et al. 2002; Elnar et al. 2021) and ocean circulation (Vyushin and Kushner 2009). The associated parameters in the modulating function $g(T)$ can be attributed to factors of the environment referred to as “effective ambient temperature” (Dietrich et al. 2020) both with biotic and abiotic influence. On the other hand, sea level may be modulated by the changes in sea density as caused by temperature (thermostatic)

and/or salinity (allosteric) (Antonov et al. 2002; Munk 2003; Ishii et al. 2006). It has pointed out; however, the influence of these factors cannot be directly extracted from our model rather we suspect that these environmental parameters have direct implications for the modulation of the HI and SL dynamics as asserted in Barkhordarian et al. (2012).

The long memory characteristics of the model presented herein have an important implication to predicting both HI and SL in larger timescales (Rypdal 2015), thus the decadal prediction of temperature rise (Hansen et al. 2006) as related to the latter can be well described. Since both HI and SL exhibit the same stochastic behavior, analyses can be simplified over these events using Equation 4 above including their interrelatedness. Also, analyzing interrelated events with a unified stochastic model offered more reliable analyses, reducing the degree of errors whereby employing only a few scaling factors. It is presented in this paper that both HI and SL exhibited the same stochastic model with memory, and thus can be analyzed singly using Equation 4. These events' long memory ranges ($\mu > 1$) are often good in predicting their changes over extended durations. Similarly, this unified stochastic model provides more reliable analyses reducing the degree of errors as such only a few scaling factors can be employed.

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ETHICAL CONSIDERATIONS

This research does not have any animal nor human subject. It utilized secondary data under the principles of data science and analyses of big data describing dynamics of heat index and sea level as temperature-dependent climatic systems. The use of accessible data was properly cited in the manuscript.

DECLARATION OF COMPETING INTEREST

The authors declare that there are no competing interests to any authors. The use of heat - index and sea level data were accompanied with appropriate citations.

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ROLE OF AUTHORS: LRD - Heat index data acquisition and model fitting. Main conception of dynamical model for heat index data; JS - Sea level data acquisition and model fitting. Main conception of dynamical model for heat index data; ARE - Manuscript preparation - rationalization and discussion part; KPC - Manuscript preparation - review of literature and theoretical consideration part; GM - completion of publishable paper; data analysis and unification of dynamical model; corresponding author.